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ε'/ε from Lattice QCD

M. Ciuchini^a, E. Franco^b, L. Giusti^c, V. Lubicz^a, G. Martinelli^{b,†}

^a Dipartimento di Fisica, Università di Roma Tre and INFN, Sezione di Roma Tre
Via della Vasca Navale 84, I-00146 Rome, Italy

^b Dipartimento di Fisica, Università di Roma “La Sapienza” and INFN, Sezione di Roma,
P.le A. Moro 2, I-00185 Rome, Italy

^c Department of Physics, Boston University, Boston, MA 02215 USA.

Abstract

Lattice calculations of matrix elements relevant for kaon decays, and in particular for ε'/ε , are reviewed. The rôle of the strange quark mass is also discussed. A comparison with other non-perturbative approaches used to compute kaon decay amplitudes is made.

† Presented by G. Martinelli.

1 Introduction

Theoretical predictions for non-leptonic decays are obtained by introducing an effective low-energy Hamiltonian expressed in terms of local operators and of the corresponding Wilson coefficients. The latter can be computed in perturbation theory, whereas the matrix elements of the operators have to be evaluated within some non-perturbative approach. For kaon decays, the Wilson coefficients are known at the next-to-leading order accuracy [1]–[8] and the main uncertainties come from the calculation of the matrix elements. In this talk we review the present status of lattice computations of matrix elements which are relevant in kaon decays. We focus, in particular, on those which enter the calculation of ε'/ε and make also a comparison with other methods, namely the Chiral Quark Model (χ QM) and the large N expansion [9]–[14]. For a more general discussion of the theory of CP violation in kaon decays see [15] and references therein.

Some general remarks are necessary before entering a more detailed discussion. Given the large numerical cancellations which may occur in the theoretical expression of ε'/ε , a solid prediction should avoid the “Harlequin procedure”. This procedure consists in patching together B_6 from the χ QM, B_8 from the $1/N$ expansion, $m_s^{\overline{MS}}$ from the lattice, etc., or any other combination/average of different methods. All these quantities are indeed strongly correlated (for example B_6 and B_8 in the $1/N$ expansion or B parameters and quark masses in the lattice approach [16]) and should be consistently computed within each given theoretical framework. Unfortunately, none of the actual non-perturbative methods is in the position to avoid completely the Harlequin procedure, not even for the most important input parameters only. The second important issue is the consistency of the renormalization procedure adopted in the perturbative calculation of the Wilson coefficients and in the non-perturbative computation of the operator matrix elements. This problem is particularly serious for the χ QM and the $1/N$ expansion, and will be discussed when comparing the lattice approach to these methods. We will address, in particular, the problem of the quadratic divergences appearing in the $1/N$ expansion. This is an important issue, since the authors of ref. [13, 14] find that these divergences provide the enhancement necessary to explain the large values of ReA_0 and of ε'/ε .

Schematically, ε' can be cast in the form

$$\varepsilon' = \frac{\exp(i\pi/4)}{\sqrt{2}} \frac{\omega}{ReA_0} \times [\omega^{-1}ImA_2 - (1 - \Omega_{IB})ImA_0] \quad (1)$$

where $\omega = ReA_2/ReA_0$ and ReA_0 are taken from experiments, and Ω_{IB} is a correcting factor, estimated in refs. [17]–[19], due to isospin-breaking effects. Using the operator product expansion, the $K \rightarrow \pi\pi$ amplitudes ImA_2 and ImA_0 are computed from the matrix elements of the effective Hamiltonian, expressed in terms of Wilson coefficients and renormalized operators

$$\langle \pi\pi | \mathcal{H}^{\Delta S=1} | K^0 \rangle_{I=0,2} = \sum_i \langle \pi\pi | Q_i(\mu) | K^0 \rangle_{I=0,2} C_i(\mu) \quad (2)$$

where the sum is over a complete set of operators, which depend on the renormalization scale μ . Wilson coefficients and matrix elements of the operators $Q_i(\mu)$, appearing in the effective Hamiltonian, separately depend on the choice of the renormalization scale and scheme. This dependence cancels in physical quantities, such as ImA_2 and ImA_0 , up to higher-order corrections in the perturbative expansion of the Wilson coefficients. For this crucial cancellation to take place, the non-perturbative method used to compute hadronic matrix elements must allow a definition of the renormalized operators consistent with the scheme used in the calculation of the Wilson coefficients.

So far, lattice QCD is the only non-perturbative approach in which both the scale and scheme dependence can be consistently accounted for, using either lattice perturbation theory or non-perturbative renormalization techniques [20, 21]. This is the main reason why the authors of refs. [8, 22, 23, 24] have followed this approach over the years.

There is a general consensus [15] that the largest contributions are those coming from Q_6 and Q_8 (for ImA_2), with opposite sign, and sizeable contributions may come from Q_3 , Q_4 and Q_9 (for ImA_2) in the presence of large cancellations between Q_6 and Q_8 , i.e. when the prediction for ε'/ε is of $\mathcal{O}(10^{-4})$ ¹. For this reason the following discussion, and the comparison with other calculations, will be focused on the determination, and errors, of the matrix elements of the two most important operators.

¹The operator $Q_{3,4,5,6}$ only contribute to the $I = 0$ amplitudes.

2 Matrix elements from lattice QCD

The evaluation of physical $K \rightarrow \pi\pi$ matrix elements on the lattice relies on the use of Chiral Perturbation Theory (χ PT): so far only $\langle \pi | Q_i(\mu) | K \rangle$ and $\langle \pi(\vec{p} = 0) \pi(\vec{q} = 0) | Q_i(\mu) | K \rangle_{I=2}$ (with the two pions at rest) have been computed for a variety of operators. The physical matrix elements are then obtained by using χ PT at the lowest order. This is a consequence of the difficulties in extracting physical multi-particle amplitudes in Euclidean space-time [25]. Proposals to overcome this problem have been presented, at the price of introducing some model dependence in the lattice results [26]. The use of χ PT implies that large systematic errors may occur in the presence of large corrections from higher-order terms in the chiral expansion and/or from FSI. This problem is common to all approaches: if large higher-order terms in the chiral expansion are indeed present and important, any method aiming to have these systematic errors under control must be able to reproduce the FSI phases of the physical amplitudes. The approaches of ref. [9, 10] and [12, 13, 14, 27], however, give FSI smaller than their physical values.

$I = 2$ matrix element of Q_8 . There exists a large set of quenched calculations of $\langle Q_8 \rangle_2$ performed with different formulations of the lattice fermion actions (Staggered, Wilson, tree-level improved, tadpole improved) and renormalization techniques (perturbative, boosted perturbative, non-perturbative), at several values of the inverse lattice spacing $a^{-1} = 2 \div 3$ GeV [16, 21, 28, 29, 30]. All these calculations, usually expressed in terms of $B_8^{(3/2)}$, give consistent results within 20% of uncertainty. Among the results, in the numerical estimates presented in sect. 5, we have taken the central value from the recent calculation of ref. [16], where the matrix elements $\langle Q_8 \rangle_2$ and $\langle Q_7 \rangle_2$ have been computed without any reference to the quark masses, and inflated the errors to account from the uncertainty due to the quenched approximation (unquenched results are expected very soon) and the lack of extrapolation to zero lattice spacing. For a discussion on the rôle of quark masses see below at the end of this section.

Matrix element of Q_6 . For $\langle Q_6 \rangle_0$ from the lattice, the situation appears worse today than a few years ago when the calculations of refs. [8, 22, 23] were performed:

- i) until 1997, the only existing lattice result, obtained with staggered

fermions (SF) without NLO lattice perturbative corrections, was $B_6 = 1.0 \pm 0.2$ [31]. This is the value used in previous analyses [8, 22, 23];

ii) with SF even more accurate results have been quoted recently, namely $B_6 = 0.67 \pm 0.04 \pm 0.05$ (quenched) and $B_6 = 0.76 \pm 0.03 \pm 0.05$ (with $n_f = 2$) [32];

iii) $\mathcal{O}(\alpha_s)$ corrections, necessary to match lattice operators to continuum ones at the NLO, are so huge for Q_6 in the case of SF (in the neighbourhood of -100% [33]) as to make all the above results unreliable. Note, however, that the corrections tend to diminish the value of $\langle Q_6 \rangle_0$;

iv) the latest lattice results for this matrix element, computed with domain-wall fermions [34] from $\langle \pi | Q_6 | K \rangle$, are absolutely surprising: $\langle Q_6 \rangle_0$ has a sign opposite to what expected in the VSA, and to what is found with the χ QM and the $1/N$ expansion. Moreover, the absolute value is so large as to give $\varepsilon'/\varepsilon \sim -120 \times 10^{-4}$. Were this confirmed, even the conservative statement by Andrzej Buras [15], namely ... *that certain features present in the Standard Model are confirmed by the experimental results. Indeed the sign and the order of magnitude of ε'/ε predicted by the SM turn out to agree with the data...* would result too optimistic. In order to reproduce the experimental number, $\varepsilon'/\varepsilon \sim 20 \times 10^{-4}$, not only new physics is required, but a large cancellation should also occur between the Standard Model and the new physics contributions. Since this result has been obtained with domain-wall fermions, a lattice formulation for which numerical studies started very recently, and no details on the renormalization and subtraction procedure have been given, we consider premature to use the value of the matrix element of ref. [34] in phenomenological analyses. Hopefully, new lattice calculations will clarify this fundamental issue.

B parameters and quark masses. Following the common lore, matrix elements of weak four-fermion operators are given in terms of the so-called B -parameters which measure the deviation of their values from those obtained in the Vacuum Saturation Approximation (VSA). A classical example is provided by the matrix element of the $\Delta S = 2$ left-left operator $Q^{\Delta S=2} = \bar{s}\gamma_\mu(1 - \gamma_5)d \bar{s}\gamma^\mu(1 - \gamma_5)d$ relevant to the prediction of the CP-violation parameter ε

$$\langle \bar{K}^0 | Q^{\Delta S=2} | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 B_K . \quad (3)$$

VSA values and B -parameters are also used for matrix elements of operators entering the expression of ε'/ε , in particular $Q_6 = \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\beta \sum_q \bar{q}_\beta \gamma_\mu (1 + \gamma_5) q_\alpha$ and $Q_8 = 3/2 \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\beta \sum_q e_q \bar{q}_\beta \gamma_\mu (1 + \gamma_5) q_\alpha$

$$\begin{aligned} \langle \pi\pi | Q_6(\mu) | K^0 \rangle_{I=0} &= -4 \left[\frac{M_{K^0}^2}{m_s(\mu) + m_d(\mu)} \right]^2 (f_K - f_\pi) B_6(\mu) \\ \langle \pi\pi | Q_8(\mu) | K^0 \rangle_{I=2} &= \sqrt{2} f_\pi \left[\left(\frac{M_{K^0}^2}{m_s(\mu) + m_d(\mu)} \right)^2 \right. \\ &\quad \left. - \frac{1}{6} (M_K^2 - M_\pi^2) \right] B_8^{(3/2)}(\mu) . \end{aligned} \quad (4)$$

Since in the VSA and in the $1/N$ expansion the expression of the matrix elements is quadratic in $m_s + m_d$, predictions for the physical amplitudes are heavily affected by the specific value which we assume for this quantity. Contrary to f_K , M_K , quark masses are not directly measured by experiments and the present accuracy in their determination is still rather poor [35, 36]. Therefore, the “conventional” parametrization induces a large systematic uncertainty in the prediction of the physical amplitudes of $\langle Q_6 \rangle_{I=0}$ and $\langle Q_8 \rangle_{I=2}$ (and of any other left-right operator). Moreover, whereas for $Q^{\Delta S=2}$ we introduce \hat{B}_K as an alias of the matrix element, by using (4) we replace each of the matrix elements with 2 unknown quantities, i.e. the B -parameter and $m_s + m_d$. Finally, in many phenomenological analyses, the values of the B -parameters of $\langle Q_6 \rangle_{I=0}$ and $\langle Q_8 \rangle_{I=2}$ and of the quark masses are taken by independent lattice calculations, thus increasing the spread of the theoretical predictions. All this can be avoided in the lattice approach, where matrix elements can be computed from first principles. In ref. [16] a new parametrisation of the matrix elements in terms of well known experimental quantities, without any reference the strange (down) quark mass, has been introduced. This results in a determination of physical amplitudes with smaller systematic errors. The interested reader can refer to [16] for details.

Before ending this discussion, we wish to illustrate the correlation existing between the B parameters and the quark masses in lattice calculations. On the lattice, quark masses are often extracted from the matrix elements of the (renormalized) axial current (A_μ) and pseudoscalar density ($P(\mu)$) (for

simplicity we assume degenerate quark masses)

$$m(\mu) \equiv \frac{1}{2} \frac{\langle \alpha | \partial_\mu A_\mu | \beta \rangle}{\langle \alpha | P(\mu) | \beta \rangle}, \quad (5)$$

where α and β are physical states (typically α is the vacuum state and β the one-pseudoscalar meson state) and $m(\mu)$ and $P(\mu)$ are renormalized in the same scheme. On the other hand, the B parameters of Q_6 and Q_8 are obtained (schematically) from the ratio of the following matrix elements, evaluated using suitable ratios of correlation functions ²:

$$B_{6,8}(\mu) \propto \frac{\langle \pi | Q_{6,8}(\mu) | K \rangle}{\langle \pi | P_\pi(\mu) | 0 \rangle \langle 0 | P_K(\mu) | K \rangle}, \quad (6)$$

where P_π and P_K are the pseudoscalar densities with the flavour content of the pion or kaon, respectively. Eqs. (5) and (6) demonstrate the strong correlation existing between B parameters and quark masses: large values of the matrix elements of $P(\mu)$ correspond, at the same time, to small values of $m(\mu)$ and $B_{6,8}(\mu)$. Physical amplitudes, instead, behave as

$$\langle Q_{6,8} \rangle = \text{const.} \times \frac{B_{6,8}(\mu)}{m(\mu)^2}, \quad (7)$$

where “const.” is a constant which may be expressed in terms of measurable quantities (specifically M_K and f_K) only. From eqs. (5) and (6), we recognize that the dependence on $\langle P(\mu) \rangle$ cancels in the ratio $B_{6,8}/m(\mu)^2$, appearing in the physical matrix elements.

Previous lattice studies preferred to work with B parameters because these are dimensionless quantities, not affected by the uncertainty due to the calibration of the lattice spacing. This method can still be used, provided that quark masses and the B parameters from the same simulation are presented together (alternatively one can give directly the ratio $B_{6,8}/m(\mu)^2$). In ref. [16], two possible definitions of dimensionless “ B parameters”, which can be directly related to physical matrix elements without using the quark masses have been proposed.

The strange quark mass. Although in lattice calculations of matrix elements any reference to quark masses can be avoided, these are fundamental

² See for example ref. [21]. For simplicity the superscript $(3/2)$ in B_8 is omitted.

parameters of the Standard Model and are used in the large N expansion. Here we would like to add only a few remarks to ref. [36], where this subject has been reviewed.

The first observation is the following. Lattice calculations that use non-perturbative renormalization methods obtain the quark masses without errors coming from the truncation of the perturbative series (typically in the RI-MOM or the Schrödinger renormalization schemes; for a complete set of references see ref. [36]). The conversion of these results to the “standard” \overline{MS} scheme can be done at the N³LO. Differences between NLO, N²LO and N³LO are important, $\sim 6 \div 10$ MeV for $m_s^{\overline{MS}}$, as demonstrated by the following example

NLO	N ² LO	N ³ LO
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$$m_\ell^{\overline{MS}}(2\text{ GeV}) = \{5.2(5); 4.9(5); 4.8(5)\} \text{ MeV},$$

$$m_s^{\overline{MS}}(2\text{ GeV}) = \{120(9); 114(9); 111(9)\} \text{ MeV}, \quad (8)$$

taken from ref. [37]. Therefore, when confronting results from different calculations it is necessary to specify the order at which the results have been obtained. In table 1 of ref. [36], results obtained with perturbation theory at NLO or with non-perturbative methods at the N³LO are directly compared. This, for the reasons discussed above, is misleading. We also note that in most of the phenomenological applications, for example with QCD sum rules, the theoretical expressions are only known at the NLO and, for consistency, quark masses at the same level of accuracy should be used.

By comparing the results obtained with the non-perturbatively improved action at $\beta = 6.2$ (corresponding to $a^{-1} \sim 2.6$ GeV, which is the value used by the APE Collaboration) with those extrapolated to the continuum (table 2 of ref. [38]), one finds the discretization errors at this value of the lattice spacing and with this action to be $3 \div 4\%$. This is much smaller than the 15% quoted in [36].

The large reduction of the value of the masses in the unquenched case, found by the CP-PACS Collaboration, is not confirmed by other lattice calculations by the MILC [39] and APE Collaborations [40] and it is at variance with the bounds of ref. [41]. We think that further investigation is required on this important point.

3 Renormalization group invariant operators

Wilson coefficients and renormalized operators are usually defined in a given scheme (HV , NDR , RI), at a fixed renormalization scale μ , and depend on the renormalization scheme and scale. This is a source of confusion in the literature. Quite often, for example, one finds comparisons of B parameters computed in different schemes. Incidentally, we note that the NDR scheme used in the lattice calculation of ref. [29] differs from the standard NDR scheme of refs. [3]–[8]; on the other hand, the HV scheme of ref. [3] is not the same as the HV scheme of ref. [7]. In some cases, the differences between different schemes may be numerically large, e.g. $B_8^{(3/2)HV} \sim 1.3 B_8^{(3/2)NDR}$ at $\mu \sim 2$ GeV. To avoid all these problems, it is convenient to introduce a Renormalization Group Invariant (RGI) definition of Wilson coefficients and composite operators which generalises what is usually done for B_K using the RGI B -parameter \hat{B}_K . The idea is very simple. Physical amplitudes can be written as

$$\langle F|\mathcal{H}|I\rangle = \langle F|\vec{Q}(\mu)|I\rangle \cdot \vec{C}(\mu), \quad (9)$$

where $\vec{Q}(\mu) \equiv (Q_1(\mu), Q_2(\mu), \dots, Q_N(\mu))$ is the operator basis and $\vec{C}(\mu)$ the corresponding Wilson coefficients, represented as a column vector. $\vec{C}(\mu)$ is expressed in terms of its counter-part, computed at a large scale M , through the renormalization-group evolution matrix $\hat{W}[\mu, M]$

$$\vec{C}(\mu) = \hat{W}[\mu, M]\vec{C}(M). \quad (10)$$

The initial conditions for the evolution equations, $\vec{C}(M)$, are obtained by perturbative matching of the full theory, which includes propagating heavy-vector bosons (W and Z^0), the top quark, SUSY particles, etc., to the effective theory where the W , Z^0 , the top quark and all the heavy particles have been integrated out. In general, $\vec{C}(M)$ depends on the scheme used to define the renormalized operators. It is possible to show that $\hat{W}[\mu, M]$ can be written in the form

$$\hat{W}[\mu, M] = \hat{M}[\mu]\hat{U}[\mu, M]\hat{M}^{-1}[M], \quad (11)$$

with

$$\hat{U}[\mu, M] = \left[\frac{\alpha_s(M)}{\alpha_s(\mu)} \right]^{\hat{\gamma}_Q^{(0)T}/2\beta_0}, \quad \hat{M}[\mu] = \hat{1} + \frac{\alpha_s(\mu)}{4\pi} \hat{J}[\lambda(\mu)], \quad (12)$$

where $\hat{\gamma}_Q^{(0)T}$ is the leading order anomalous dimension matrix and $\hat{J}[\lambda(\mu)]$ can be obtained by solving the Renormalization Group Equations (RGE) at the NLO. By defining

$$\hat{w}^{-1}[\mu] \equiv \hat{M}[\mu] [\alpha_s(\mu)]^{-\hat{\gamma}_Q^{(0)T}/2\beta_0} , \quad (13)$$

we get

$$\hat{W}[\mu, M] = \hat{w}^{-1}[\mu] \hat{w}[M] . \quad (14)$$

The effective Hamiltonian (9) can then be written as

$$\begin{aligned} \mathcal{H} &= \vec{Q}(\mu) \cdot \vec{C}(\mu) = \vec{Q}(\mu) \hat{W}[\mu, M] \vec{C}(M) \\ &= \vec{Q}(\mu) \hat{w}^{-1}[\mu] \cdot \hat{w}[M] \vec{C}(M) = \vec{Q}^{RGI} \cdot \vec{C}^{RGI} , \end{aligned} \quad (15)$$

with

$$\vec{C}^{RGI} = \hat{w}[M] \vec{C}(M) , \quad \vec{Q}^{RGI} = \vec{Q}(\mu) \cdot \hat{w}^{-1}[\mu] . \quad (16)$$

\vec{C}^{RGI} and \vec{Q}^{RGI} are scheme and scale independent at the order at which the Wilson coefficients have been computed.

4 Comparison with other methods

In this section, we briefly discuss the relevant aspects which distinguish the lattice approach from others which have been used in the literature to predict ε'/ε .

The original approach of the Munich group was to extract the values of the relevant matrix elements from experimental measurements [4, 6]. This method guarantees the consistency of the operator matrix elements with the corresponding Wilson coefficients. Unfortunately, with the Munich method it is impossible to get the two most important contributions, namely those corresponding to $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$. For this reason, “guided by the results presented above and biased to some extent by the results from the large- N approach and lattice calculations”, the authors of ref. [42] have taken $B_6 = 1.0 \pm 0.3$ and $B_8^{(3/2)} = 0.8 \pm 0.2$, and $m_s^{\overline{MS}} = 110 \pm 20$ MeV at $\mu = 1.3$ GeV. These values, if assumed to hold in the HV regularization, are very close to those used in ref. [24]. They do not come however from a calculation

consistently made within a given theoretical approach (large N expansion, χ QM or lattice for example).

The $1/N$ expansion and the χ QM are effective theories. To be specific, in the framework of the $1/N$ expansion the starting point is given by the chiral Lagrangian for pseudoscalar mesons expanded in powers of masses and momenta. At the leading order in $1/N$, local four-fermion operators can be written in terms of products of currents and densities, which are expressed in terms of the fields and coupling of the effective theory. In higher orders, a (hard) cutoff, Λ_c , must be introduced to compute the relevant loop diagrams. The cutoff is usually identified with the scale at which the short-distance Wilson coefficients must be evaluated.

Divergences appearing in factorizable contributions can be reabsorbed in the renormalized coupling of the effective Lagrangian and in the quark masses, non-factorizable corrections constitute the part which should be matched to the short distance coefficients. By using the intermediate colour-singlet boson method, the authors of refs. [11]–[14],[43] claim to be able to perform a consistent matching, including the finite terms, of the matrix elements of the operators in the effective theory to the corresponding Wilson coefficients. It is precisely this point which, in our opinion, has never been demonstrated in a convincing way. If the matching is “consistent”, then it should be possible to show analytically that the cutoff dependence of the matrix elements computed in the $1/N$ expansion cancels that of the Wilson coefficients, at least at the order in $1/N$ at which they are working. Moreover, if really finite terms are under control, it should be possible to tell whether the coefficients should be taken in HV , NDR or any other renormalization scheme.

The fact that in higher orders even quadratic divergences appear, with the result that the logarithmic divergences depend now on the regularization, makes the matching even more problematic. Theoretically, we cannot imagine any mechanism to cancel the cutoff dependence of the physical amplitude in the presence of quadratic divergences, which should, in our opinion, disappear in any reasonable version of the effective theory. It is also important to show (and to our knowledge it has never been done) that the numerical results for the matrix elements are stable with respect to the choice of the ultraviolet cutoff. This would also clarify the issue of the routing of the mo-

menta in divergent integrals. For example, the matrix elements in the meson theory could be computed in some lattice regularization.

5 Numerical results

As discussed above, all the methods used in the calculation of ε'/ε are not completely satisfactory and in general suffer from large theoretical uncertainties.

The lattice approach can, in principle, compute the relevant matrix elements without any model assumption (at least at the lowest order in the chiral expansion), and with operators consistently defined to match the Wilson coefficients of the effective Hamiltonian. In spite of these advantages the lattice results for $\langle Q_6 \rangle_0$ are inconclusive, as discussed before. Regarding the surprising result of ref. [34], we think that further scrutiny and confirmation from other calculations are needed before using it in a phenomenological analysis.

In the absence of any definite result for $\langle Q_6 \rangle_0$ from the lattice, ref. [24] assumed for this matrix element the value

$$\langle Q_6 \rangle_0 \equiv \langle \pi\pi | Q_6^{HV} | K^0 \rangle_{I=0} = -0.4 \pm 0.4 \text{ GeV}^3, \quad (17)$$

and

$$\langle Q_5 \rangle_0 = 1/3 \langle Q_6 \rangle_0, \quad (18)$$

at a scale $\mu = 2 \text{ GeV}$. The value of the matrix element in eq. (17) corresponds to $B_6 = 1.0 \pm 1.0$ for a “conventional” mass fixed to $m_s^{\overline{MS}} + m_d^{\overline{MS}} = 130 \text{ MeV}$.

For $\langle Q_{7,8} \rangle_2$, the values of ref. [16] (obtained with an improved action using non-perturbatively renormalized operators at $\mu = 2 \text{ GeV}$) have been used, namely

$$\langle Q_7 \rangle_2 \equiv \langle \pi\pi | Q_7^{HV} | K^0 \rangle_{I=2} = 0.18 \pm 0.06 \text{ GeV}^3, \quad (19)$$

$$\langle Q_8 \rangle_2 \equiv \langle \pi\pi | Q_8^{HV} | K^0 \rangle_{I=2} = 0.62 \pm 0.12 \text{ GeV}^3, \quad (20)$$

where the superscript HV denotes the t’Hooft-Veltman renormalization scheme.

By varying the input parameters as described in ref. [24] and by weighting the Monte Carlo events with the experimental constraints, the prediction for ε'/ε is

$$\varepsilon'/\varepsilon = (3.6_{-6.3}^{+6.7} \pm 0.5) \times 10^{-4}, \quad (21)$$

where the third error on ε'/ε is an estimate of the residual scheme dependence due to unknown higher-order corrections in the perturbative expansion³. Given the large theoretical uncertainties, and taking into account some differences in the calculation of this quantity (choice of the renormalization scale, values of several B parameters, etc.), the result in eq. (21) is in substantial agreement, though slightly lower, with the recently upgraded evaluation of ref. [42]: $\varepsilon'/\varepsilon = (7.7_{-3.5}^{+6.0}) \times 10^{-4}$ and $\varepsilon'/\varepsilon = (5.2_{-2.7}^{+4.6}) \times 10^{-4}$ in NDR and in HV respectively. It is also very close to previous estimates of the Rome [22, 23] and Munich group [4, 6]. This agreement it is not surprising since the two groups use very similar inputs for the matrix elements and the experimental parameters have only slightly changed in the last few years. The crucial question, namely a quantitative determination of $\langle Q_6 \rangle_0$, remains unfortunately still unsolved.

All the above results are, however, much lower than the recent measurements of KTeV, $Re(\varepsilon'/\varepsilon) = (28.0 \pm 4.1) \times 10^{-4}$, of NA48, $Re(\varepsilon'/\varepsilon) = (18.5 \pm 7.3) \times 10^{-4}$, or than the present world average $Re(\varepsilon'/\varepsilon)_{WA} = (21.2 \pm 4.6) \times 10^{-4}$, determined by the results of refs. [44]–[46],[47]

By scanning various input parameters (in the conventional approach B_6 , $B_8^{(3/2)}$, $\alpha_s(M_Z)$, $Im\lambda_t$ etc.) and in particular by choosing them close to their extreme values it is possible to obtain $\varepsilon'/\varepsilon \sim 20 \times 10^{-4}$. This also gives the impression of a better agreement (lesser disagreement) between the theoretical predictions and the data. For example, in ref. [24] the scanning gives $-11 \times 10^{-4} \leq \varepsilon'/\varepsilon \leq 27 \times 10^{-4}$. In spite of the fact that the experimental world average is compatible with the “scanned” range above, a conspiracy of several inputs in the same direction is necessary in order to get a large value of ε'/ε . For central values of the parameters, the predictions are, in general, much lower than the experimental results. For this reason, barring the possibility of new physics effects [48], we believe that an important message is

³ The value in eq. 21 is slightly different from that presented at the the Conference and quoted by ref.[15]. The reason is that the final analysis of ref. [16] found for $\langle Q_8 \rangle_{I=2}$ a value larger by about 15% than the preliminary one.

arriving from the experimental results:

penguin contractions (or eye-diagrams, not to be confused with penguin operators), which are usually neglected within factorization, give contributions which makes the matrix elements definitely larger than their factorized values.

This implies that the “effective” B parameters of the relevant operators, specifically those relative to the matrix elements of Q_1 and Q_2 for $Re(A_0)$ and of Q_6 for ε'/ε are much larger than 1. This interpretation would provide a unique dynamical mechanism to explain both the $\Delta I = 1/2$ rule and a large value of ε'/ε [49]. Large contributions from penguin contractions are actually found by calculations performed in the framework of the Chiral Quark Model (χ QM) [9]–[10] or the $1/N$ expansion [11, 13, 14, 27]. It is very important that these indications find quantitative confirmation in other approaches, for example in lattice QCD calculations. Note that naïve explanations of the large value of ε'/ε , such as a very low value of $m_s^{\overline{MS}}$, would leave the $\Delta I = 1/2$ rule unexplained.

Finally, one may try to quantify the amount of enhancement required for the matrix element of Q_6 in order to explain the experimental value of ε'/ε . A fit of $\langle Q_6 \rangle_0$ to $Re(\varepsilon'/\varepsilon)_{WA}$ gives $\langle Q_6 \rangle_0 = -1.2^{+0.25}_{-0.21} \pm 0.15 \text{ GeV}^3$, about 2.5 times larger than the central value used in our analysis.

References

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